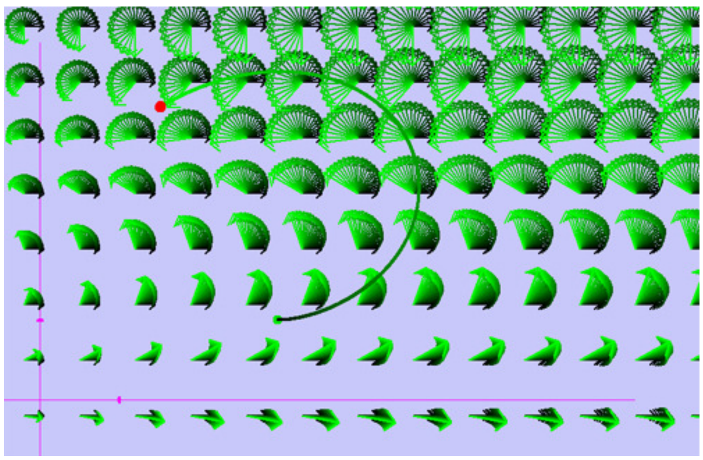
John Long's Hilbert Space

The "vectors" in the original linear space are *Zeno contours*: contours in the complex plane that are given *parametrically* as $z(t)$ or $\gamma(t)$, $0 \le t \le 1$, as solutions of the differential equations $\frac{dz}{dt} = f(z,t) - t$, where $f(z,t)$ is an underlying time-dependent vector field, or *algorithmically* as $z_{k,n} = z_{k-1,n} + \frac{1}{n}\left(f(z_{k-1,n}, \tfrac{k}{n}) - z_{k-1,n}\right)$ with $k : 1 \to n$ and $n \to \infty$. Assume $f(z,t)$ is continuous in $C \times [0,1]$.



Define an *inner product* that provides an abstract version of *orthogonality* (intersection of two lines at right angles) : $\langle \gamma_1, \gamma_2 \rangle = \int_0^1 \gamma_1 \overline{\gamma_2}\, dt$, from which one infers a *norm* of the space ("how far" the vector is from the origin): $\|\gamma\| = \sqrt{\langle \gamma, \gamma \rangle}$. This norm induces a *metric* for the space (defines "distance between vectors") : $d(\gamma_1, \gamma_2) = \|\gamma_1 - \gamma_2\|$.

Linear operators are employed on Hilbert Spaces. A very simple example is: $O(\gamma) = e^{i\frac{\pi}{4}}\gamma$.

Comment: There is nothing original here . . . simply an elementary example of a Hilbert Space for an aging rock climber and writer who loves them. Physicists use Hilbert Spaces as computational tools and employ somewhat different notations.